

Arithmetic and Geometric Sequences (2.1) Homework

1. The first three terms of an arithmetic sequence are $a_1 = 11$, $a_2 = 7$, and $a_3 = 3$.
 - a. Find the common difference.
 - b. Find a_{20} .

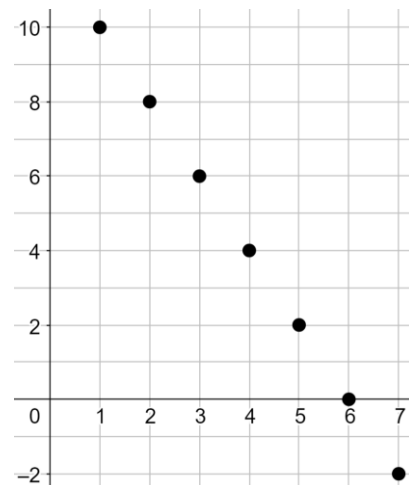
2. The second term of an arithmetic sequence is $a_2 = 10$. The third term of that sequence is $a_3 = 15$.
 - a. Find the first term of the sequence.
 - b. Find the common difference.
 - c. Find an expression for the n th term of the sequence.
 - d. Find a_{20} .

3. The second term of a geometric sequence is $g_2 = 10$. The third term of that sequence is $g_3 = 15$.
 - a. Find the first term of the sequence.
 - b. Find the common ratio.
 - c. Find an expression for the n th term of the sequence.

4. The fifth term of a geometric sequence is $g_5 = 48$. The eighth term of the sequence is $g_8 = 162$.
 - a. Find the common ratio.
 - b. Find an expression for the n th term of the sequence.

5. The fifth term of an arithmetic sequence is $a_5 = 48$. The eighth term of the sequence is $a_8 = 162$.
 - a. Find the common difference.
 - b. Find an expression for the n th term of the sequence.

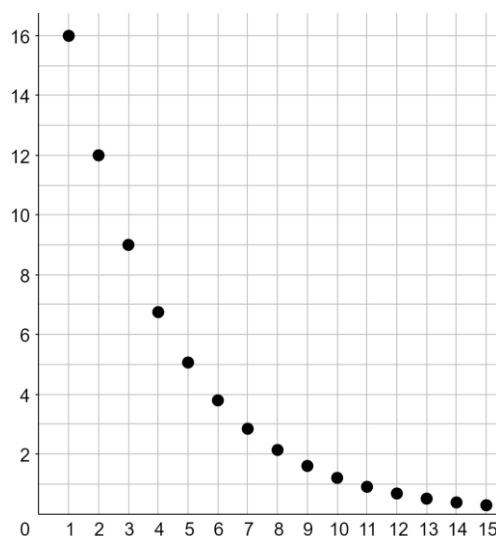
6. Values of an arithmetic sequence a_n are graphed in the figure. Find a_{20} and find an expression for the n th term of the sequence.



7. Luke has to read *Catch-22* for his AP Language and Composition course. His copy of the book is 453 pages long. On day 1, he has 453 pages to read. On day 2, he only has 412 pages left. On day 3, he only has 371 pages left.
- If the number of pages he has left each day forms an arithmetic progression, construct a formula for the number of pages Luke has left on day n . (Note: Luke is a mad man who does not mind stopping mid-chapter)
 - What is a reasonable domain restriction for the formula from part a? In other words, for what values of n does it make sense to use the formula?
8. Rose is thinking of an arithmetic sequence with first term x and common difference 28. Poe is thinking of a geometric sequence with first term x and common ratio 2. Incredibly, the fourth term of Rose's sequence and the fourth term of Poe's sequence are equal! Find x .

9. Values of a geometric sequence g_n are graphed in the figure. Which one of the following gives the general term of this sequence?

- $g_n = 16\left(\frac{3}{4}\right)^n$
- $g_n = 32\left(\frac{1}{2}\right)^n$
- $g_n = 16\left(\frac{3}{4}\right)^{n-1}$
- $g_n = 32\left(\frac{1}{2}\right)^{n-1}$



10. An arithmetic sequence has terms $a_{20} = 18$ and $a_{21} = 26$.
- Find the general formula for a_n .
 - Find a_{10} .
11. An arithmetic sequence has terms $a_4 = 20$ and $a_{28} = -52$.
- Find the general formula for a_n .
 - Find a_{10} .
12. A geometric sequence has terms $g_{16} = 30$ and $g_{19} = 60$.
- Find the general formula for g_n .
 - Find g_{10} .
13. A sequence has seventh term 8 and ninth term 50.
- If this sequence is arithmetic, find the value of the eighth term.
 - If this sequence is geometric with a positive common ratio, find the value of the eighth term.
 - Why did part b say “with a positive common ratio”? What would have been the issue with that question if that phrase were omitted?
14. An arithmetic sequence has $a_7 = 10$ and $a_{15} = 22$. Find the first term of the sequence.
15. The first term of an arithmetic sequence is $a_1 = 78$ and the common difference is $d = -3$. Find the smallest value of n for which a_n is negative.

Arithmetic and Geometric Sequences (2.1) HW Answers

1.

a. $d = -4$

b. $a_{20} = -65$

2.

a. $a_1 = 5$

b. $d = 5$

c. $a_n = 5 + 5(n - 1)$

OR

$$a_n = 5n$$

d. $a_{20} = 100$

3.

a. $g_1 = \frac{20}{3}$

b. $r = \frac{3}{2}$

c. $g_n = \frac{20}{3} \left(\frac{3}{2}\right)^{n-1}$

OR

$$g_n = 10 \left(\frac{3}{2}\right)^{n-2}$$

OR

$$g_n = 15 \left(\frac{3}{2}\right)^{n-3}$$

4.

a. $r = \frac{3}{2}$

b. $g_n = 48 \left(\frac{3}{2}\right)^{n-5}$

OR

$$g_n = 162 \left(\frac{3}{2}\right)^{n-8}$$

OR

$$g_n = \frac{512}{81} \left(\frac{3}{2}\right)^n$$

OR

$$g_n = \frac{256}{27} \left(\frac{3}{2}\right)^{n-1}$$

5.

a. $d = 38$

b. $a_n = 48 + 38(n - 5)$

OR

$$a_n = 162 + 38(n - 8)$$

OR

$$a_n = -142 + 38n$$

OR

$$a_n = -104 + 38(n - 1)$$

6. $a_n = 10 - 2(n - 1)$

$a_{20} = -28$

7.

a. $a_n = 453 - 41(n - 1)$

b. This formula applies for positive integer values of n . Further, it only applies for $n \leq 12$. Otherwise, we might conclude that on day 13, Luke has -39 pages left to read, which would be nonsensical. In other words, the only values in the domain should be $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

8. $x = 12$

Reasoning: The fourth term of Rose's sequence is $a_4 = x + 28 \cdot 3$. The fourth term of Poe's sequence is $g_4 = x(2)^3$. Since we are told these are equal, we know...

$$x + 28 \cdot 3 = x(2)^3$$

$$x + 84 = 8x$$

$$84 = 7x$$

$$x = 12$$

9. C

10.

a. $a_n = 18 + 8(n - 20)$

b. $a_{10} = -62$

11.

a. $a_n = 20 - 3(n - 4)$

b. $a_{10} = 2$

12.

a. $g_n = 30(\sqrt[3]{2})^{n-16}$

b. $g_{10} = \frac{15}{2}$

13.

a. $a_8 = 29$

b. $g_8 = 20$

c. With only $g_7 = 8$ and $g_9 = 50$, there are two possible common ratios. $r = \frac{5}{2}$ and $r = -\frac{5}{2}$ are both possible. The phrase "positive common ratio" narrowed it down to only one possibility.

14. $a_1 = 1$

15. $n = 28$ (since $a_{26} = 3$, $a_{27} = 0$, $a_{28} = -3, \dots$)